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The quadrupole moments of the first 2^+ states of ^{106}Cd and ^{108}Cd

I Hall, M F Nolan, D J Thomas and M J Throop†

Oliver Lodge Laboratory, University of Liverpool, Oxford Street, Liverpool L69 3BX, UK

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Abstract. The static electric quadrupole moments of the first 2^+ states of ^{106,108}Cd have been measured by Coulomb excitation. The measured moments are significantly smaller than previously reported values and indicate a gradual change in quadrupole moment across both the isotopes of Cd and the $N = 58$ and 60 isotones.

1. Introduction

The properties of the low-lying states of the doubly even nuclei in the $Z \simeq 50$ region may be fairly well described in terms of the vibrational model. In the last few years, however, the static quadrupole moments of the first 2^+ (one phonon) states of several of these nuclei have been measured in precision Coulomb excitation experiments and in many cases these moments have been found to be large. In the simple vibrational model these moments are zero.

We have been making a systematic study of the quadrupole moments in this mass region. The results for ^{106}Cd and ^{108}Cd reported here are discussed in § 3 in relation to our earlier results, to other published measurements among the Cd isotopes, and to the particle–vibrator coupling model.

2. Experiment

The static quadrupole moment of an excited state has a second-order effect on the Coulomb excitation of that state, the effect being approximately proportional to the mass of the bombarding ion. Thus for each of the two nuclei we essentially compared the excitation probability of the first 2^+ state measured with ^{16}O bombarding ions to that measured with ^{4}He ions. The experimental method and analysis have been described in some detail previously (Thomas *et al* 1973) and only a brief account will be given here.

Targets enriched to 88% ^{106}Cd and 79% ^{108}Cd were bombarded with 36 MeV ^{16}O ions and 9·5 MeV ^{4}He ions from the Liverpool University EN Van de Graaff. The targets were prepared by evaporation of cadmium on to backings of copper (for ^{16}O bombardment) or carbon (for ^{4}He bombardment); these backings were thick enough to stop the recoiling excited nuclei. The target thicknesses were approximately 600 μg cm^{-2} and were measured in each case by reversing the target and finding the change in energy of the ions scattered back from the backing. In the analysis the effective bombarding

† Present address: Physics Department, University of Oregon, Eugene, Oregon 97403, USA.

energy was taken to be the incident ion energy less half the energy lost in traversing the target. For the actual energy losses in these experiments and the known energy dependence of cross sections and stopping powers this is a very good approximation.

The $2^+ \rightarrow 0^+$ decay γ rays were counted in coincidence with ions scattered through a mean angle of 162° into an annular silicon detector. The γ rays were counted in a 40 cm^3 Ge(Li) detector placed at an angle of 66° to the beam direction and 3·7 cm from the target. To make an accurate comparison of the ^{16}O- and ^{4}He-induced yields it is essential to maintain a constant detection efficiency. Thus the γ ray detector was precisely located and for each measurement the ^{16}O and ^{4}He bombardments were carried out within the same accelerator run to avoid any long-term change in Ge(Li) detector efficiency.

A conventional fast–slow coincidence electronics system was used and coincidence events were sorted in time and γ ray energy using an on-line computer. Spectra were gated by a window set in the particle spectrum around the particles scattered from cadmium. A gated γ ray spectrum is shown in figure 1. The γ ray resolution, typically about 5 keV, was compromised by the pulse shaping necessary to avoid excessive pile-up at high rates. This was the most significant cause of count-rate loss and the latter was determined experimentally by taking measurements over a wide range of beam currents. The coincidence yield was taken to be the $2^+ \rightarrow 0^+$ photopeak count normalized to the number of backscattered particles and the final measured quantity was the ratio of the

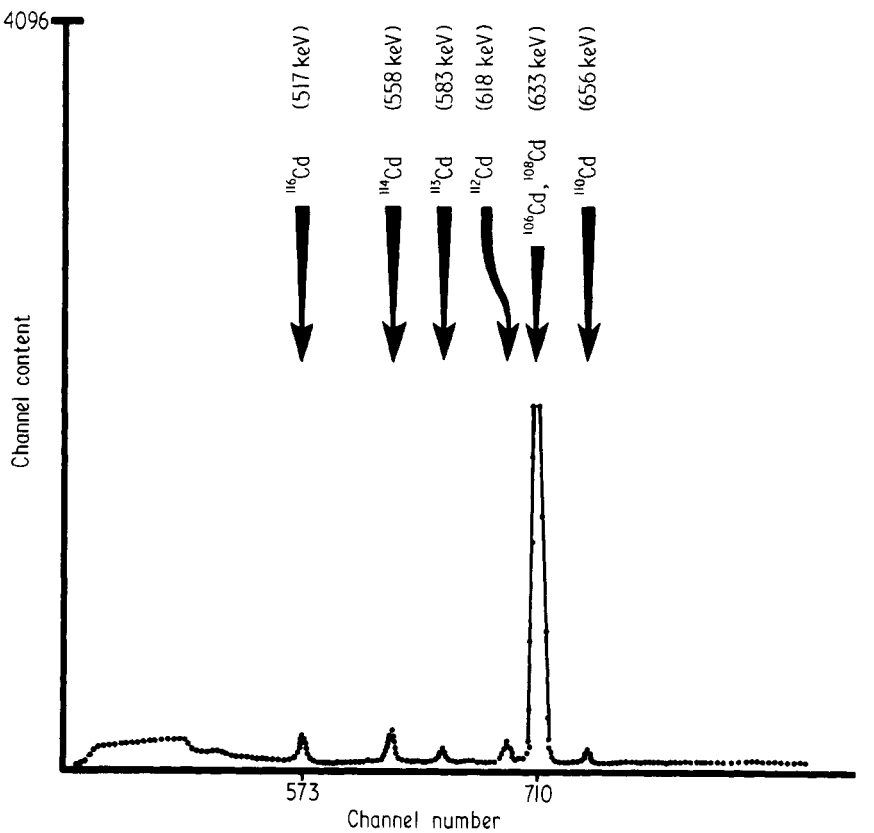

Figure 1. Coincidence γ ray spectrum induced by 36 MeV ^{16}O ions on ^{106}Cd. The lines indicated are all ground state transitions.

coincidence yield induced by ^{16}O ions to that by ^{4}He. We had two runs on ^{106}Cd and one on ^{108}Cd and the measured yield ratios are given in table 1. The $2^+ \rightarrow 0^+$ γ rays have very nearly the same energy in ^{106}Cd and ^{108}Cd; thus in the ratios given in table 1 very small corrections have been applied to allow for the presence of some ^{106}Cd in the nominal ^{108}Cd target and vice versa.

Table 1. Ratios of the coincidence yield of the $2^+ \rightarrow 0^+$ decay γ ray measured with ^{16}O ions to that measured with ^{4}He

Target	Effective energy of bombarding ion (MeV)		Yield ratio
	^{16}O	^{4}He	
^{106}Cd	34·94	9·37	6·82 ± 0·12
	34·94	9·44	6·54 ± 0·14
^{108}Cd	34·99	9·43	6·52 ± 0·09

These data were analysed in the manner described by Thomas *et al* (1973) using the de Boer–Winther Coulomb excitation computer program. The energy levels included in the analysis are shown in figure 2 and the values of the E2 matrix elements used are given in table 2. These are based on the B(E2) values reported by Milner *et al* (1969). The relative phases of these matrix elements are unknown and are unimportant except for those connecting the second 2^+ state ($2'^+$), the ground state, and the first 2^+ state. The $2'^+$ state gives rise to a second order interference term in the Coulomb excitation of the first 2^+ state which involves the product of the matrix elements coupling the 0^+, 2^+ and $2'^+$ states. The sign of this term is unknown but its effect can be found by changing the sign of just one of the above matrix elements. Thus the analysis for each nucleus was carried out with alternative signs of the matrix element M_{14} (see table 2). The results for

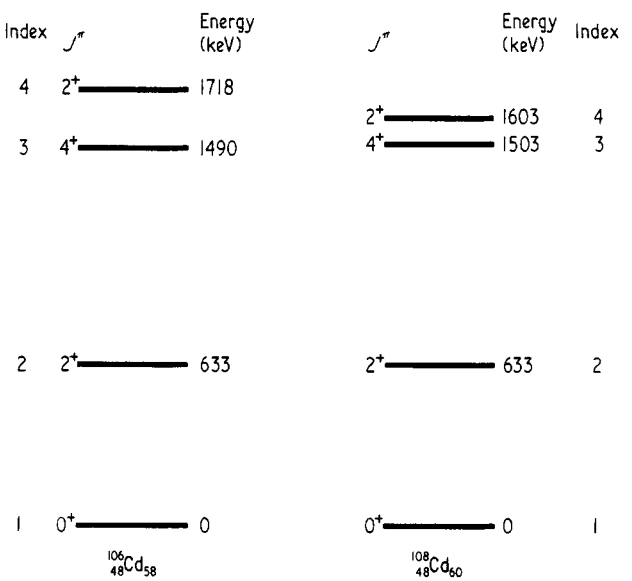

Figure 2. The energy levels of ^{106}Cd and ^{108}Cd included in the analysis.

Table 2. Reduced E2 matrix elements (in eb) used in the analysis

(*a*) ^{106}Cd				
Level	1	2	3	4
1	0	−0·65	0	∓0·19
2	−0·65	M_{22}	−1·05	−0·50
3	0	−1·05	0	0
4	∓0·19	−0·50	0	0
(*b*) ^{108}Cd				
Level	1	2	3	4
1	0	−0·67	0	∓0·17
2	−0·67	M_{22}	−1·06	−0·53
3	0	−1·06	0	0
4	∓0·17	−0·53	0	0

The matrix elements are defined by

$$M_{rs} = \langle s \| i^{\lambda} \mathcal{M}(E\lambda) \| r \rangle$$

where $\mathcal{M}(E\lambda)$ is the multipole operator and $\lambda = 2$.

$$M_{rs}^2 = (2I_r + 1)B(E2, r \rightarrow s)$$

and the quadrupole moment of the 2^+ state is given by $Q_2 = -0{\cdot}758 M_{22}$.

the quadrupole moments of the first 2^+ states of ^{106}Cd and ^{108}Cd are given in table 3. The quoted errors are due mainly to counting statistics and a smaller contribution from uncertainty in target thickness. Effects of uncertainties in the values of the matrix elements given in table 2 were examined and found to be negligible in comparison.

Table 3. Deduced quadrupole moments Q_2 of the first 2^+ states of ^{106}Cd and ^{108}Cd. Two measurements were made for ^{106}Cd and one for ^{108}Cd

	Sign of M_{14}	Q_2 (eb)	
^{106}Cd	−	−0·13 ± 0·15 −0·17 ± 0·17	−0·15 ± 0·11
	+	+0·03 ± 0·15 −0·02 ± 0·17	+0·01 ± 0·11
^{108}Cd	−		−0·35 ± 0·13
	+		−0·20 ± 0·13

3. Discussion

In previous measurements on the isotopes of Pd and Te (Christy *et al* 1970, Harper *et al* 1971) we found that the magnitude of the quadrupole moments of the first 2^+ states increased across the sets of isotopes with increasing transition probability $B(E2, 0^+ \rightarrow 2^+)$ and decreasing level energy ΔE. This is shown graphically for the Pd isotopes in figure 3.

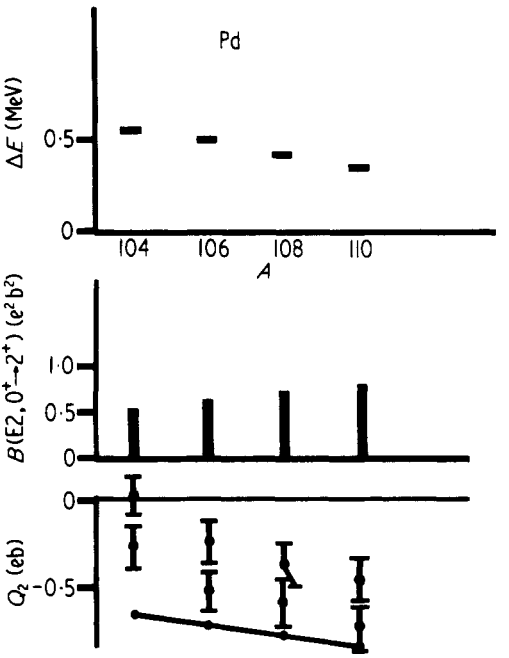

Figure 3. The variation of the quadrupole moment of the first 2^+ state across the Pd isotopes. The two alternative results for each nucleus correspond to both possible phases of the $2'^+$ interference. The line links the rotational model values $Q_2 = 0{\cdot}91\sqrt{B(\mathrm{E2}, 0^+ \rightarrow 2^+)}$. The various results of other workers given in the compilation of Christy and Häusser (1973) are not included in this diagram but they are consistent with the trend indicated by our results.

In a simple view these data are consistent with a vibrator–rotor transition and the possibility of deformed nuclei existing some way from the stability line in this mass region remains an open and interesting question.

Although a comprehensive theoretical account of all the measured quadrupole moments in this region has not yet been given, the application of the particle–vibrator coupling model has had some success (Alaga *et al* 1967, Lopac 1970, Sips 1971). The nuclei in this region have proton numbers close to the $Z = 50$ major shell closure and neutron numbers ranging across the $50 < N < 82$ shell. Thus this model, in which the few protons (or proton holes) are coupled to a harmonic vibrational core, may be expected to be appropriate. For a nonzero particle–core coupling strength the eigenstates of the total hamiltonian contain basis vibrational states of more than one phonon number and, for example, the mixing of a two-phonon component into the largely one-phonon nature of the first 2^+ state gives a nonzero static quadrupole moment for this state. Thus although the extra-core protons in appropriate shell model states contribute only a relatively small quadrupole moment, they polarize the core, thus inducing a further and larger moment of the same sign. In the particular case of Sn ($Z = 50$) the very small measured quadrupole moments are easily understood. For other nuclides this model is able to give quadrupole moments of approximately the right magnitude whilst also reproducing quite well the level energies and transition rates. It is also worth noting that, among the energy levels thus calculated for Te, Lopac (1970) drew attention to a $0^+2^+4^+6^+8^+$ sequence of states with enhanced $B(\mathrm{E2})$ values. If this is an embryo

rotational band it fits in rather nicely with the simple interpretation above of our quadrupole moment data.

The situation regarding the measured moments in the Cd isotopes is summarized in figure 4. The data comprise:

(i) the present results for ^{106}Cd and ^{108}Cd;
(ii) results of the Rutgers group (see below);
(iii) averages of all other results given in the compilation of Christy and Häusser (1973).

The results (ii) are from two experiments. Steadman *et al* (1970), using targets of natural Cd, measured the quadrupole moments of the 2^+ states of the isotopes of Cd relative to that of ^{114}Cd; for the 2^+ state of ^{114}Cd they took $Q_2 = -0{\cdot}40$ eb. In this measurement ^{106}Cd and ^{108}Cd were not resolved and a lumped value of $Q_2 = -0{\cdot}84 \pm 0{\cdot}28$ eb was

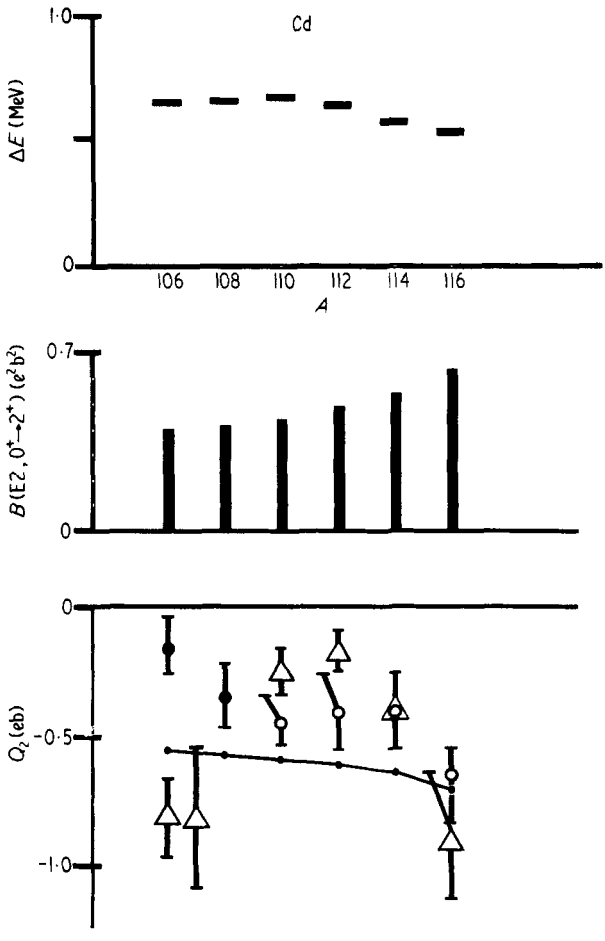

Figure 4. As for figure 3 but for the Cd isotopes. For simplicity only the quadrupole moments deduced assuming constructive $2'^+$ interference are shown. (Those deduced assuming destructive $2'^+$ interference are somewhat smaller in magnitude.) This is the phase which is given by the phonon-mixing model and which fits best the several data on ^{114}Cd. The measurements are shown in three groups as explained in the text. ● Liverpool, △ Rutgers, ○ others.

given and this is plotted between mass numbers 106 and 108 in figure 4. In a second experiment using an enriched ^{106}Cd target Kleinfeld *et al* (1970) measured $Q_2 = -0{\cdot}83 \pm 0{\cdot}16$ eb.

According to the Rutgers results above the variation of Q_2 across the Cd isotopes is considerable. At the extremes the values of $|Q_2|$ are larger than the rotational model values and there is a minimum in $|Q_2|$ at about ^{112}Cd. Steadman *et al* (1970) suggested that this minimum is due to closure of the $g_{7/2}$ neutron subshell at $N = 64$. If the neutron subshell effect is as significant as this then the utility of the particle–vibrator coupling model is put in doubt. However, we note that this effect did not manifest itself in our previous Pd data; in fact $|Q_2|$ is largest at $N = 64$ in the Pd isotopes (^{110}Pd, figure 3). Furthermore our results for ^{106}Cd and ^{108}Cd together with the other results (iii) are consistent with a variation of Q_2 according to the simple pattern mentioned at the beginning of this section.

The same trend may be seen in the variation of Q_2 across the $N = 58$ and $N = 60$ sets of isotones (figure 5) when our present results for ^{106,108}Cd are compared with our earlier measurements on ^{104,106}Pd (Christy *et al* 1970) and ^{102,104}Ru (Nolan *et al* 1973). We have included on this figure the Rutgers result for ^{106}Cd but not for ^{108}Cd

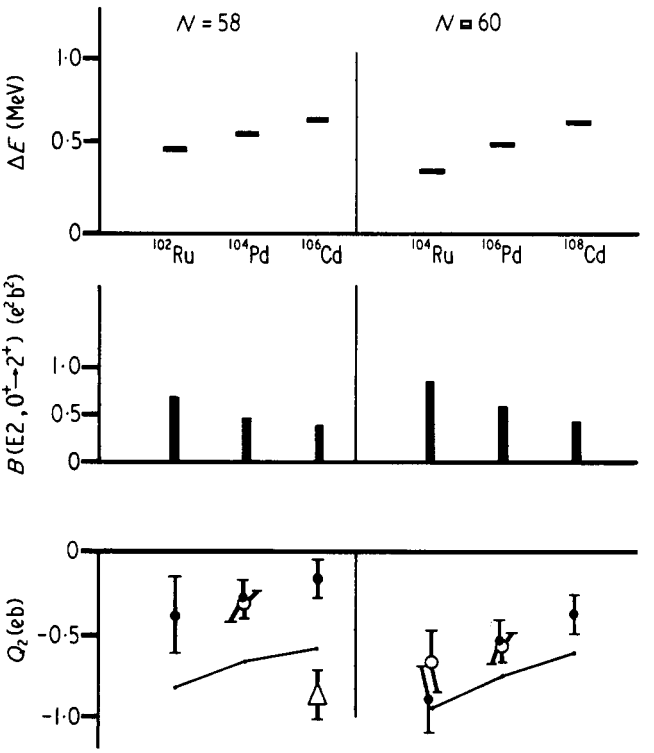

Figure 5. As for figure 4 but for the $N = 58$ and $N = 60$ isotones. Other results for Q_2 have been included in this diagram: ^{104}Pd (Ward *et al* 1971), ^{106}Pd (Lutz *et al* 1972) and ^{104}Ru (Stelson 1968)†.

† Private communication to de Boer J and Eichler J 1968 *Adv. Nucl. Phys.* 1 1–65.

since from their results

$$Q_2(^{106,108}\text{Cd lumped}) = -0{\cdot}84 \pm 0{\cdot}28 \text{ eb}$$

and

$$Q_2(^{106}\text{Cd}) = -0{\cdot}83 \pm 0{\cdot}16 \text{ eb}$$

one can infer only an imprecise result

$$Q_2(^{108}\text{Cd}) = -0{\cdot}86 \pm 0{\cdot}69 \text{ eb}.$$

In conclusion we may say that the results of Steadman *et al* (1970) and Kleinfeld *et al* (1970), which would not be readily explained by the particle–vibrator coupling model, are not supported by our data. Although it is tempting to say that our results are more likely on account of the systematic variation across isotopes and isotones, it would be circuitous to assert that these trends are thus proven. More and independent measurements are desirable.

Acknowledgments

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